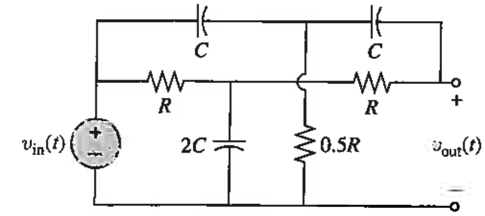

Figure P15.59 Twin-T *RC* circuit.

Step and Impulse Response

60. Consider the circuit of figure P15.60.

(a) Find the transfer function $H(s)$ and the impulse response $h(t)$.
(b) Tabulate $h(t)$ for $t = 0, 1, 2, \ldots, 8$. Use this table to do the next three parts, assuming that $v_C(0^-) = 0$.
(c) If $v_{in}(t) = 32\delta(t-2)$ V, find $v_{out}(5)$.
(d) If $v_{in}(t) = 64\delta(t-2) + 16\delta(t-5) + 2\delta(t-8)$ V, find $v_{out}(8^+)$.
(e) If the input is a rectangular pulse of duration 1 ms and amplitude 1000 V, and if it is applied at $t = 0$ s, find the values of $v_{out}(t)$ at $t = 1, 2,$ and 3 s. Compare with part (b). Note that in this *RC* circuit, the pulse width is much smaller than the circuit time constant; hence to the circuit this rectangular pulse looks like an impulse, $\delta(t)$.

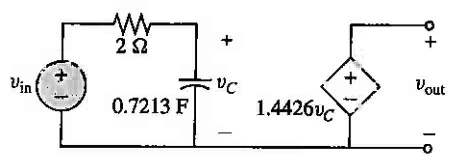

Figure P15.60

61. Consider the circuit of figure P15.61.

(a) Find the transfer function $H(s)$ and the impulse response $h(t)$.
(b) Find the response due to a unit step function input by two different methods:

i. Inverse Laplace transformation of $V_{out}(s)$.
ii. Integration of $h(t)$.

(c) Find the response due to a unit ramp function input by two different methods:

i. Inverse Laplace transformation of $V_{out}(s)$.
ii. Integration of the unit step response found in part (b).

(d) Find the maximum value of $|v_{out}(t)|$ due to an input $v_{in}(t) = 200u(t)$ V. Does the answer surprise you?

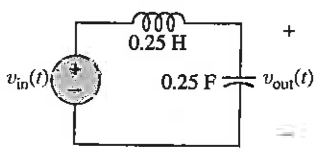

Figure P15.61

62. Consider the circuit of figure P15.62a.

(a) Find the transfer function $H(s)$ and the impulse response $h(t)$.
(b) Find the step response.
(c) Find $v_{out}(t)$ due to a rectangular pulse input, as shown in figure P15.62b, with $\Delta = 2/LC$. *Hint:* Make use of the result of part (b).
(d) Find $v_{out}(t)$ in part (c) when $\Delta \to 0$. *Note:* The result should agree with part (a), since $i_{in}(t)$ approaches $\delta(t)$.

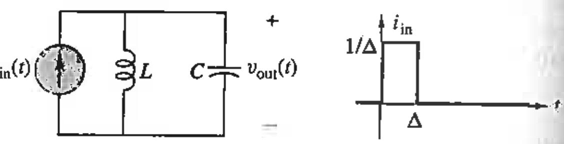

Figure P15.62

63. (a) Suppose the zero-state step response of the circuit of figure P15.63 is $i_{out}(t) = e^{-2t}u(t)$ A. Find the zero-state response to $i_{in}(t) = 2\delta(t-1)$ A.
(b) Compute the zero-state response for $t \geq 0$ to a ramp input $4r(t)$. Recall that the ramp function is the integral of the step function.

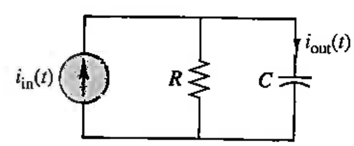

Figure P15.63

64. For the circuit of figure P15.64, suppose $i_{in}(t) = Ku(t)$ A. $v_C(0^-) \neq 0$, and the response of the circuit for $t > 0$ is $v_C(t) = 2.5u(t) + 2.5e^{-0.2t}u(t)$ V. Find the values of K and $v_C(0^-)$.

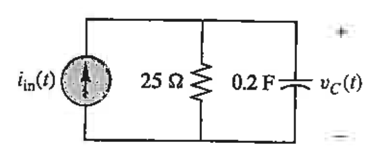

Figure P15.64

65. Suppose the impulse response of the network in figure P15.65 is

$$v_{out}(t) = [2e^{-t} + 3\cos(2t) - 1.5\sin(2t)]u(t) \text{ V}.$$

(a) Compute the transfer function $H(s)$.
(b) Find all poles and zeros, and draw the pole–zero plot.
(c) If there is no initial stored energy in the network, i.e., no initial conditions, compute the network response to $v_{in}(t) = \sin(t)u(t)$ V. Does your answer make sense? Why or why not?

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ANSWERS
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to a ramp

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$v_2(t)$
(b) I
$v_3(t)$
(c) I
$v_4(t)$
68. For t
transform
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Find $H_1($
(a)
(b)

Inital- a

69. (a)

Find

(d) Now suppose $v_{in}(t) = \cos(2t)u(t)$ V. For large t, show that $v_{out}(t) \approx Kt\cos(2t + \phi)$ V for appropriate K and ϕ. Does the response remain finite as $t \to \infty$? Is the network stable or unstable?

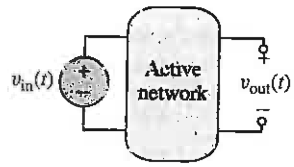

Figure P15.65

66. A time shift differentiator circuit has the property that its zero-state output is always equal to the derivative of its input shifted in time by some $T > 0$. Hence its impulse response is $h(t) = \delta'(t - T)$, a shifted version of the derivative of the delta function. Suppose $T = 1$ and the input to the circuit is given by $f(t)$, as sketched in figure P15.66. Compute the zero-state response $y(t)$ of the circuit at $t = 2.5$ s and $t = 6$ s.

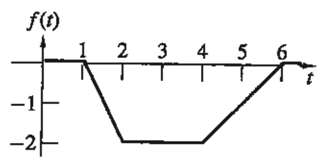

Figure P15.66

ANSWERS: −2, 1

67. The Laplace transform of the response of a relaxed circuit to a ramp $v_1(t) = r(t - 1)$ V is

$$Y(s) = \frac{e^{-2s}}{s^3}$$

(a) Find the response of this same relaxed circuit to an input $v_2(t) = u(t)$ V.
(b) Find the response of this same relaxed circuit to an input $v_3(t) = \delta(t)$ V.
(c) Find the response of this same relaxed circuit to an input $v_4(t) = 5\delta(t - 2)$ V.

68. For this problem you are to use the properties of Laplace transforms as listed in chapter 13. Suppose the Laplace transform of the impulse response $h(t)$ of a circuit is given by

$$H(s) = \frac{s+2}{(s+1)(s+3)}$$

Find $H_1(s)$ and $H_2(s)$ when:
(a) $h_1(t) = h(4t)$,
(b) $h_2(t) = e^{-4t}h(4t - 8)$.

Inital- and Final-Value Problems

69. (a) The Laplace transform of a signal $f(t)$ is

$$\frac{(5s-1)(4s-5)(6s-2)}{s(2s+1)(3s+2)(5s+4)}$$

Find the value of $f(0^+)$.

(b) Repeat part (a) when the Laplace transform of a signal $f(t-1)u(t-1)$ is

$$e^{-s}\frac{(10s-1)(8s-5)(12s-2)}{s(2s+1)(3s+2)(5s+4)}$$

ANSWERS: 4, 32.

70. The output $v_{out}(t)$ of a particular circuit is engineered to track a reference signal $v_{ref}(t)$. After the circuit overheated, it was found that

$$V_{ref}(s) - V_{out}(s) = \frac{1}{s} - \frac{1}{s}\ln\left(\frac{s+1}{s+0.2865}\right)$$

Find the difference $v_{ref}(t) - v_{out}(t)$ for large t.

ANSWER: −0.25.

71. Given the following functions $F_i(s)$, find $f_i(0^+)$ and $f(\infty)$ for those cases where the initial-value and final-value theorems are applicable (a root-finding program is useful in this endeavor):

(a) $F_1(s) = \dfrac{21s+5}{0.1s^2+4s+3}$.

(b) $F_2(s) = \dfrac{2s^3+7s^2+s+4}{s(s^3+s^2+7s+6)}$.

(c) $F_3(s) = \dfrac{s^2+4s+3}{s(s^4+5s^3+5s^2+4s+4)}$.

72. (a) If $F(s) = (e^{-s}/s)\ln[(2.7183s+1)/(s+3)]$, find $f(1^+)$.
(b) If $F(s) = (e^{-2s}/s)\ln[(s+3)/(0.135335s+1)]$, find $f(2^+)$.

ANSWERS: 1, 2.

73. The capacitor voltages $V_C(s)$ of various networks have Laplace transforms as given. Use only the initial- and final-value theorems to determine $v_C(0^+)$ and $v_C(\infty)$ for each of the transforms. If either theorem is not applicable, explain why.

(a) $\dfrac{(14s-1)^2}{s^2(7s+2)}$.

(b) $\dfrac{(2s-15)(2-e^{-6s})}{s(s+10)(1-e^{-3s})}$.

(c) $200\dfrac{(0.1s-1)(0.2s+1)}{s(s^2+144)}$.

(d) $\dfrac{16s-1}{(2s+1)^2}\left[1+\dfrac{4s-1}{2s+1}+\left(\dfrac{2s-1}{2s+1}\right)^2+\left(\dfrac{4s-1}{2s+1}\right)^3\right]$.

74. The capacitor voltages $V_C(s)$ of various networks have Laplace transforms as given. Use only the initial- and final-value theorems to determine $v_C(0^+)$ and $v_C(\infty)$ for each of the transforms. If either theorem is not applicable, explain why.

(a) $20\dfrac{0.1s-1}{s(2s+25)}$.

(b) $\dfrac{(6s+1)(3s+1)(2-e^{-20s})}{s^2(4-e^{-2s})}$.

(c) $\dfrac{4}{s} - \dfrac{e^{-\sqrt{s}}}{s(s+0.5)}$.

(d) $\dfrac{Ke^{-2/s}}{s}$.